Indian Statistical Institute
Mid-Semestral Examination 2007-2008
M.Math I Year
Algebra

Time: 3 hrs | Date:10/03/2008 | Total Marks : 100

Instructor : N S N Sastry

Answer all Questions.Your answers should be logically complete.

1. Let R be a commutative ring with 1; $P_1, \cdots, P_n$ be ideals in R such that R/P_i is a Noetherian ring for $i = i, \cdots, n$. Assume that $\bigcap_i^n P_i = \{0\}$. Show that $R[X_1, \cdots, X_m]$ is Noetherian for all $m \geq 1$. [8 + 12]

2. Let R be a commutative ring with 1 and A be an ideal in R. Let M be a finitely generated R-module such that AM=M. Show that there exists an element r in R such that $rM = 0$ and $1 - r \in A$. [15]

3. Define a primary R-module associated with a prime ideal P of R. If $A_1, \cdots, A_n$ are primary R-modules associated with P , then so is their intersection $A_1 \bigcap \cdots \bigcap A_n$. [3 + 12]

4. Define the R-module $M \bigotimes_R$ N if M and N are R-modules. If M,N and N' are R-modules , then show that $(M \bigotimes_R N) \bigotimes_R N'$ is isomorphic to $M \bigotimes_R (N \bigotimes_R N')$ as R-modules. [15]

5. Let k be an algebraically closed field. Along with relavent definitions, prove that there exists an inclusion reversing bijective correspondence between the radical ideals of $k[X_1, \cdots, X_n]$ and the closed algebraic subsets of k^n. [20]

6. (a)If A,B and P are ideals in a ring R with $AB \subseteq P$ and P a prime ideal of R , then show that either $A \subseteq P$ or $B \subseteq P$. [5]

   (b) Define the localization R_P of a ring R and the localization M_P of an R-module M at a prime ideal P of R . Show that M_P and $M \bigotimes_R R_P$ are isomorphic as R_P-module. [10]

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